Resonance and Vibration Isolation Experiment

Background: A spring and mass system shown in Figure-1 usually represents a physical model of a machine supported by a flexible base, like amortisors, elastic rubbers, unrigid base itself etc, where **m** is mass of the machine, and **k** and **c** denote the equivalent spring proportionality constant, and and damping coefficient of the base, respectively. **F(t)** is the exciting force field created by centrifugal effects of unbalanced rotating shafts, and other moving unbalanced components of the machine, especially at high speed.

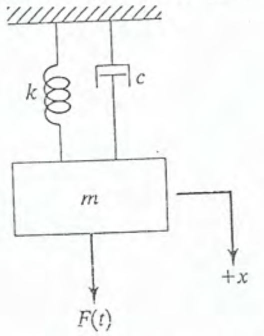

Figure-1: A spring-mass-damper system.

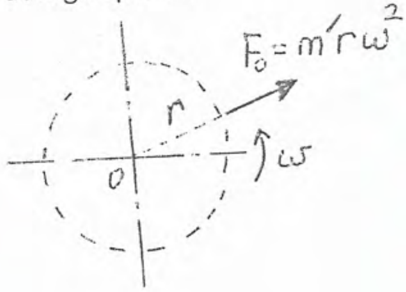

Figure-2: Centrifugal effect of unbalanced mass.

Simple behaviour of this repeated exciting force can be represented by a harmonic function $F(t)=F_0 \cos\omega t$. Notice that this exciting force **F(t)** can be created by an unbalanced mass m' with unbalanced distance **r** at constant angular velocity **ω** of the shaft, since centifugal force become $F_0=m'r\omega^2$. This exciting force causes vibrations of the system, and normalised shaking force **T** transmitted to the base. On the other hand, natural frequency ω_n of this system is known as tendency of oscillations with amplitutes without exciting force after an initial disturbance.

$$\omega_n=(k/m)^{1/2} \quad : \text{natural frequency}$$

Maximum amplitutes occur when the frequency of exciting force **ω** matches or approaches to the natural frequency ω_n. This phenomena is called **resonance**, that occurs at $\omega=\omega_n$, where the system undergoes dangerously large oscillations. If the angular velocity of motor is required to be close to the natural frequency, then resonance phenomena has to be prevented using various methods, two of which will be demonstrated in this experiment.

Aim: Aim of this experiment is to demonstrate two approaches to prevent resonance, one of which is to shift natural frequency of the system, and the other one is adding a vibration absorber to the system.

Method of Shifting Natural Frequency of the System:

A spring and mass system in Figure-1 is a single degree of vibrating system such that its position can be precisely defined by a single independent parameter **X**. Normalised amplitutes of a single degree of system is shown in Figure 3, where δ_{st} is static deflection of the spring under the weight of mass $W=mg$, and **X** denotes amplitutes of vibration.

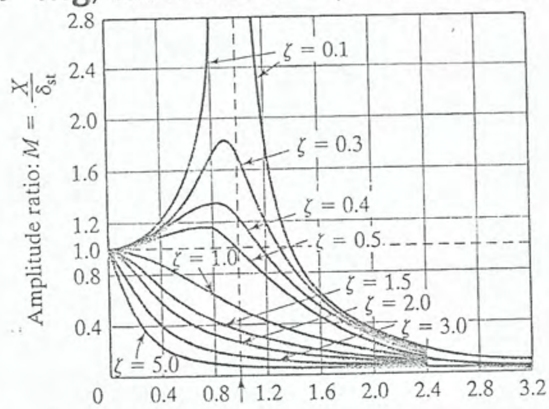

Figure-3: Variation of X with frequency ratio.

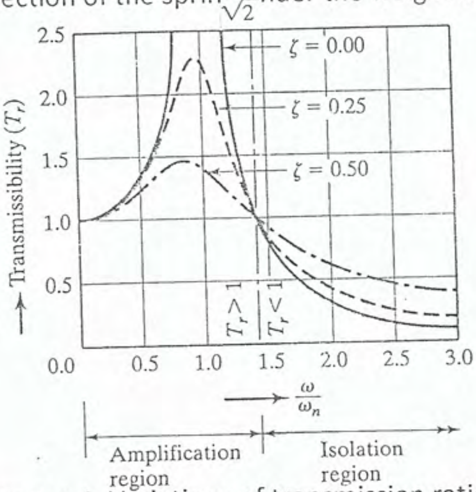

Figure-4 :Variations of transmission ratio T.

Horizontal axis (ω/ω_n) is the ratio of angular velocity ω of exciting effects (usually a motor) to the natural frequency of the system ω_n, and ζ is the damping factor of this equivalent spring effect. As expected, when there is no exciting effect $(\omega=0)$, the ratio of $(X/\delta_{st})=1$ or $x=\delta_{st}=mg/k$ for all damping factors. Notice that damping factor ζ is a normalised parameter, where its value takes place between $0<\zeta<1$ for under-damped systems, and $\zeta=0$ for an undamped system.

In the absence of damping, as angular velocity of motor increases , amplitutes of vibration increses until $\omega = \omega_n$, and then decreases after $\omega = \omega_n$. At $\omega = \omega_n$ value, amplitutes theoretically reaches infinity for undamped vibrations, that is known as **resonane**. In practice, instead of infinity, amplitutes reaches to maximum values about $\omega = \omega_n$ (in fact slightly less than ω_n) according to damping factor value. As damping increases, maximum amplitutes values become smaller.

In Figure 4, horizontal axis is the same as that in Figure 3, but vertical axis is the ratio of forces (F_t) transmitted to ground, to the amplitute of exciting force F_0. This ratio, $T=F_t/F_0$, is known as transmissibility. Similar argument can be made for Figure 4 as well, wit a difference that when $(\omega/\omega_n)=\sqrt{2}$, transmissibility become the same for any damping value. In addition, transmissibility becomes even smaller for smaller damping values. If motor has to work at resonance frequency, $\omega = \omega_n$, one approach to eliminate resonance is to shift natural frequency of the system. Since the expression of natural frequency is $\omega_n=(k/m)^{1/2}$, then increasing total mass of the system, shifts ω_n to the left side. This shift therefore prevents the resonance for this speed of the motor by means of additional masses.

Method of Adding and Absorber to the System:

An absorber is an additional spring and mass system. When an absorber is added to the actual system, whole system will have two degrees of freedom, one is the deflection of the actual mass of the motor, and the other one is the deflection of absorber mass.

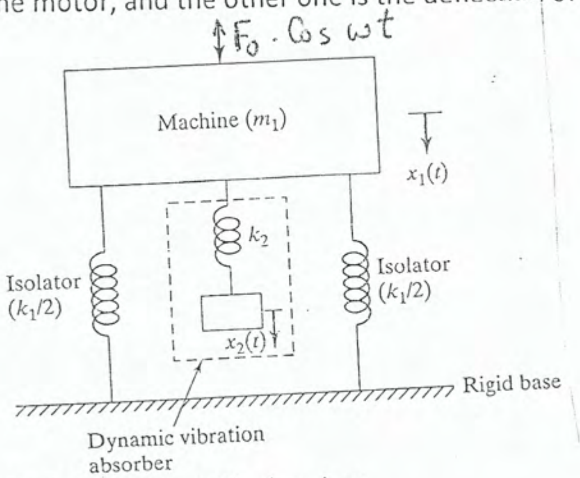

Fig 5: A system with absorber.

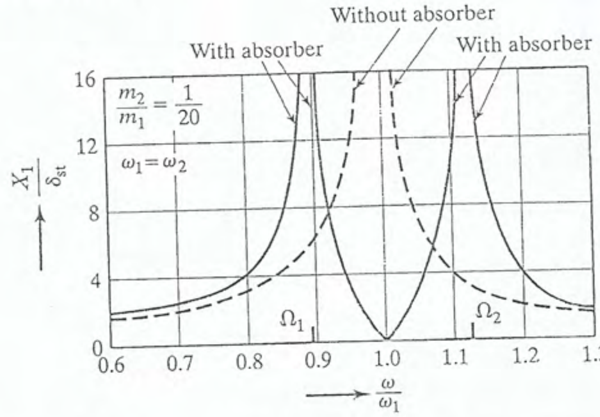

Fig 6: Effects of undamped vibration absorber on the response of the system.

When the natural frequency of absorber is properly adjusted , actual system come to rest at $\omega=\omega_n$; instead, the absorber comes to resonance.This is because, as shown in Figure 6, whole system with absorber has two natural frequencies at Ω_1 and Ω_2. Therefore at $\omega=\omega_n$, the actual system comes to rest while the absorber comes to resonance.

Experimental Set Up:

In this experiment , a **DC motor** with unbalanced mass is placed on an **elastic bar**, representing flexible base as shown in Figure 7. This experiment has two phases: **(i)** shifting natural frequency by adding extra masses, **(ii)** adding and absorber.

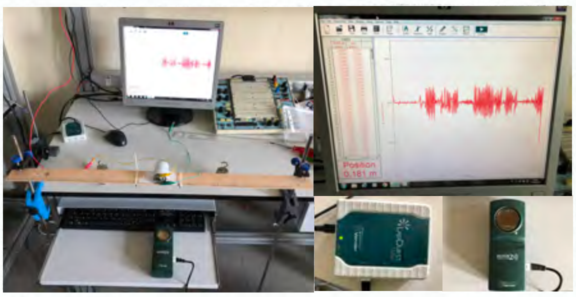

Figure 7: Experimental set up.

$$\frac{X_1}{\delta_{st}} = \frac{1 - \left(\frac{\omega}{\omega_2}\right)^2}{\left[1 + \frac{k_2}{k_1} - \left(\frac{\omega}{\omega_1}\right)^2\right]\left[1 - \left(\frac{\omega}{\omega_2}\right)^2\right] - \frac{k_2}{k_1}}$$

$$\frac{X_2}{\delta_{st}} = \frac{1}{\left[1 + \frac{k_2}{k_1} - \left(\frac{\omega}{\omega_1}\right)^2\right]\left[1 - \left(\frac{\omega}{\omega_2}\right)^2\right] - \frac{k_2}{k_1}}$$

Figure 8: Displacement formulas of system with absorber.

In order to perform the 1st phase, additional masses will be used to increase the total mass of the system, by adding them on the top of DC motor. For the 2nd phase, a **long screw** is fixed to the middle of the bar in perpendicular position. **Two bolds with adjusted masses**, which acts as masses of **absorber**, are placed on the screws in either sides. Distance of bolts to the bar can be adjusted in such a way that the equivalent spring coefficient of the absorber comes to the right value to bring the motor at rest. In addition, there is also an **electronic device** to drive the DC motor by voltage variations. A chart is also provided to relate voltage supplied by the electronic device to the angular velocity of motor in rpm.

Phase 1: Method of Shifting Natural Frequency by Adding Extra Masses:

Having started this phase, make sure that bolts with masses are to be removed from the screws. Also notice that when there is no rotation, the bar is already statically deflected δ_{st} amount under the weight of motor and the weight of bar itself. This static deflection can also be used to determine equivalent spring constant after a calibration. As angular velocity of unbalanced motor is increased by voltage increments, vibration of the bar increases until reaching large oscillations, that is resonance of the system. When ω is further increased, oscillations gets smaller as expected from Figure 3. Notice that angular velocity ω at resonance, also matches to the natural frequency that is $\omega_n=(k/m)^{1/2}$. This ω_n value can be verified using equivalent spring coefficient and equivalent mass of the system. When extra masses are added to the system to eliminate resonance, oscillations become much smaller. This is because, the natural frequency of the system is shifted to the left by the addition of extra masses. As a result, resonance causing dangerously large oscillations are prevented by shifting natural frequency of the system to the left without changing exciting effect, that is angular velocity of motor, ω.

Phase 2: Method of Adding and Absorber:

Having started eliminating resonance using an absorber, DC motor is operated at resonance frequency in the absence of bolts with masses on screws. When the bolts are mounted on screws, vibration amplitutes of bar changes because of the fact that the absorber is added to the system. The distances of bolts to the base can be adjusted in such a way that the bar with DC motor comes to the rest, but the bolts representing absorber masses start large oscillations. This is exactly the same bahaviour explained in Fig 5. It has therefore been observed that addition of an absorber can bring a spring and mass system to rest excited by resonance frequency.

Tasks:

Notice that, having started eliminating resonance using an absorber, DC motor is operated at resonance frequency initially.

(i) Determine natural frequency of the DC motor and the support bar without an absober, and observe resonance phenomena.

(ii) Obtain equivalent spring constant using static deflection δ_{st} under the assumption that no damping exists.

(iii) Obtain equivalent mass of the system using natural frequency and equivalent spring constant.

(iv) Eliminate resonance by shifting natural frequency of the system.

(v) Eliminate resonance by adding an absorber to the system.